On the Queuing Order in Hearthstone Conquest: random.org

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1 Background

Yesterday I wrote an article that intended to introduce concepts from game theory that are relevant to understand static games. I used these concepts to describe how to choose a deck (out of three) in the first round of a Bo5 setting. In my mind, the plan was to build up from the first queuing decision (which is the simplest one if you do not take into account future turns) and then build up to the Bo5 case, explaining how each additional layer was going to affect (or not) the decision that was optimal in isolation. That was the plan and, to be clear, I was going to execute the plan as I was writing the articles (that is, I had no idea what the end solution was going to be). That was the plan.

Then, several readers criticized the article. First, some complained that the article was not a comprehensive treatment. Well, sure. Second, some complained that it was so basic that it was useless to understand the entire Bo5. Well, to me it is always best to start simple and build up. And a bunch of other criticisms, many of them valid and relevant and so others less so. However, there was one that called my attention: the claim that if your opponent is randomizing decks with equal probability (that is, using the random number generator from random.org) then that strategy cannot be exploited by you, the player. This was interesting because it does not happens in a static game where the mixing probabilities depend on the payoff values (matchup probabilities). It also meant two important things: (1) my idea of building up from Bo1 to Bo5 was flawed and was not going to lead anywhere; (2) conquest Bo5 (and Bo3) is a type of game in which the payoff matrix (matchup probabilities) affect your odds of winning but those odds are identical for any possible queuing order (again, when your opponent uses random.org). Finally, if that's the case, then both players using random.org is an equilibrium of the game. So, last night I grabbed pen and paper and worked out the algebra for Bo3 and Bo5 and concluded that:

1. I was wrong in the plan developed by the first article. Understanding the equilibrium when the problem is simply two players queuing a deck is irrelevant for understanding the Bo5 solution. To be clear, if you want to understand how to think about a 3x3 static game, see what a dominated strategy is, etc, the previous article is not wrong and you may find it useful: it simply does not serve the purpose I original intended for the article.

2. In Bo3 and Bo5, if your opponent uses random.org (i.e., the opponent randomizes/mixes decks with equal probability), then the ex-ante odds of you winning the round are independent of the order in which you queue your deck so you cannot exploit your opponent's behavior. It follows from here that if you also use random.org, we have an equilibrium where both players are happy.

	Deck A	Deck B
Deck 1	ω_1	ω_2
Deck 2	ω_3	ω_4

Table 1: Mathcup probabilities for Bo3

If you want an answer to the question: "does the queuing order in Hearthstone matter?", then that's your answer and you do not need to keep reading this article. The answer is not original to me however, as a bunch of players commented that this was the case yesterday. However, when I asked "where can I see the derivations?", the answers I got were "we know this", "somebody told me", "somebody run a simulation", or a link to a Reddit post that tackled a different question. So, I decided to explain why random.org is an equilibrium in the best way I can in this article and so I hope you find it clear. The good news is that you do not need to know anything about game theory and that the manipulations are elementary (add, subtract, and multiply). However, parts of the algebra involved are not particularly insightful so what I will do is to focus on the Bo3 case and explain the thought process. If you understand the Bo3 case, then you should be able to do the annoying Bo5 case with some patience.

Analysis of the Bo3 case

As opposed to the previous article, let's do things more abstractly and spare my friends from being accused of bringing paladin. Let's called them Player 1 (row player) and Player 2 (column player) and consider the payoff matrix in Table 1.

The actual values of the matchup probabilities $(\omega_1, \omega_2, \omega_3, \omega_4)$ are irrelevant for this argument so we keep them as variables taking values in $(0, 1)$. What we know is that Player 2 randomizes decks A and B with probability $\frac{1}{2}$. What we want to prove is that the probability of winning the Bo3 series is the same for Player 1 if he/she starts with Deck 1 or Deck 2. Let's get started.

Player 1 starts with Deck 1

The probability that Player 1 wins the first match is $\frac{1}{2}(\omega_1+\omega_2)$ - recall player 2 is using random.org. After winning with Deck 1, the probability that Player 1 wins the second match using his second deck is $\frac{1}{2}(\omega_3+\omega_4)$. We conclude that

$$P(\text{player 1 wins 2-0}) = \frac{1}{4}(\omega_1+\omega_2)(\omega_3+\omega_4)\ . \tag{1.1}$$

That is the only way Player 1 can win 2-0.

Now let's think about the 2-1 cases. Suppose Player 1 loses the first match and then wins the remaining two. Player 1 loses the first match against Deck A with probability $\frac{1}{2}(1-\omega_1)$ and then wins the series by beating Deck B twice with probability $\omega_2\omega_4$. Player 1 alternatively loses the first match against Deck B with probability $\frac{1}{2}(1-\omega_2)$ and then wins the series by beating Deck A twice with probability $\omega_1\omega_3$. We conclude that:

$$P(\text{player 1 wins after losing first match}) = \frac{1}{2}(1-\omega_1)\omega_2\omega_4 + \frac{1}{2}(1-\omega_2)\omega_1\omega_3\ . \tag{1.2}$$

The other way for Player 1 to win 2-1 is by losing the second game, after wining the first one, to then win the third and final game. Well, we already know Player 1 wins the first game with probability $\frac{1}{2}(\omega_1+\omega_2)$ and then must queue Deck 2 in the second game. Losing in the second game then happens with probability $\frac{1}{2}(1-\omega_3)$ or $\frac{1}{2}(1-\omega_4)$ and then Player 1 wins the third match by either winning against Deck A (ω_3) or against Deck B (ω_4). Putting all these together,

$$P(\text{player 1 wins after losing second match}) = \frac{1}{4}(\omega_1+\omega_2)[(1-\omega_3)\omega_4+(1-\omega_4)\omega_3] \ . \tag{1.3}$$

And this is it for the case where Player 1 starts with Deck 1. We can combine the last two probabilities to obtain:

$$P(\text{player 1 wins 2-1}) = \frac{1}{2}(1-\omega_1)\omega_2\omega_4+\frac{1}{2}(1-\omega_2)\omega_1\omega_3+\frac{1}{4}(\omega_1+\omega_2)[(1-\omega_3)\omega_4+(1-\omega_4)\omega_3] \ . \tag{1.4}$$

Player 1 starts with Deck 2

The thought process for Player 1 starting with Deck 2 is identical, but let's go over it. Player 1 wins with Deck 2 with probability $\frac{1}{2}(\omega_3+\omega_4)$ and then wins with Deck 1 (in the second match) with probability $\frac{1}{2}(\omega_1+\omega_2)$, so that:

$$P(\text{player 1 wins 2-0}) = \frac{1}{4}(\omega_1+\omega_2)(\omega_3+\omega_4) \ . \tag{1.5}$$

Clearly, we can see that the expressions in (1.1) and (1.5) are the same. This is not surprising since in order to win 2-0 you have to win with both decks consecutively and so it shouldn't matter which one you use first.

Now let's think about the 2-1 cases and start with the case where Player 1 loses the first match. Player 1 now loses the first match with probability $\frac{1}{2}(1-\omega_3)$ against Deck A and with probability $\frac{1}{2}(1-\omega_4)$ against Deck B. Player 1 then wins the series by beating either Deck A or Deck B twice so that:

$$P(\text{player 1 wins after losing first match}) = \frac{1}{2}(1-\omega_3)\omega_2\omega_4+\frac{1}{2}(1-\omega_4)\omega_1\omega_3 \ . \tag{1.6}$$

This expression is not equal to the one in (1.2) for arbitrary values of $(\omega_1,\omega_2,\omega_3,\omega_4)$. It tuns out things only perfectly aligned in the end, so you have to be patient. The other way for Player 1 to win 2-1 is by losing the second game. Player 1 wins the first game with probability $\frac{1}{2}(\omega_3+\omega_4)$ and then must queue Deck 2 in the second game. Losing in the second game now happens with probability $\frac{1}{2}(1-\omega_1)$ or $\frac{1}{2}(1-\omega_2)$ and then Player 1 wins the third match by either winning against Deck A (ω_1) or against Deck B (ω_2). Putting all these together,

$$P(\text{player 1 wins after losing second match}) = \frac{1}{4}(\omega_3+\omega_4)[(1-\omega_1)\omega_2+(1-\omega_2)\omega_1] \ . \tag{1.7}$$

Again, this expression is not equal to the one in (1.3) for arbitrary values of $(\omega_1,\omega_2,\omega_3,\omega_4)$. And this is it for the case where Player 1 starts with Deck 2. We can combine the last two probabilities to obtain:

$$P(\text{player 1 wins 2-1}) = \frac{1}{2}(1-\omega_3)\omega_2\omega_4+\frac{1}{2}(1-\omega_4)\omega_1\omega_3+\frac{1}{4}(\omega_3+\omega_4)[(1-\omega_1)\omega_2+(1-\omega_2)\omega_1] \ . \tag{1.8}$$

We reached the final part of the derivation. At first sight, the expressions in (1.4) and (1.8) do not look identical, but a simple manipulation shows that they indeed are. For example, take the expression in (1.4) and add and subtract the term $\frac{1}{4}(1-\omega_4)\omega_1\omega_3$ and the term $\frac{1}{4}(1-\omega_3)\omega_2\omega_4$. Doing so and re-arranging terms leads to:

$$\begin{aligned} P(\text{player 1 wins 2-1}) = & \frac{1}{2}(1-\omega_1)\omega_2\omega_4 + \frac{1}{2}(1-\omega_2)\omega_1\omega_3 \\ & + \frac{1}{2}(1-\omega_3)\omega_2\omega_4 + \frac{1}{2}(1-\omega_4)\omega_1\omega_3 \\ & + \frac{1}{4}(\omega_2-\omega_1)(\omega_3-\omega_4) \ . \end{aligned} \tag{1.9}$$

If you now take the expression in (1.8) and add and subtract the term $\frac{1}{4}(1-\omega_1)\omega_2\omega_4$ and the term $\frac{1}{4}(1-\omega_2)\omega_1\omega_3$, you get to exactly the same expression in (1.9).

Conclusion 1: We just showed that in Bo3 conquest when you face an opponent that uses random.org, the probability that you win the series does not depend on the order in which you queue your decks (from an ex-ante perspective of course). Player 1 is indifferent between queuing Deck 1 or Deck 2. The result does not depend on the values of the matchup probabilities $(\omega_1, \omega_2, \omega_3, \omega_4)$. As promised, showing this required elementary manipulations.

Conclusion 2: Player 1 is indifferent between decks 1 and 2. However, we just learned that mixing with equal probabilities cannot be exploited, so Player 1 is better off following the same strategy and randomizing/mixing decks 1 and 2 with equal probability. We conclude that both players mixing decks with equal probability is an equilibrium of this game (as nobody has incentives to deviate from their strategies). Whether this is the unique equilibrium of this game or not is not something I looked into. But, in a way it is not that important because the one feature of this equilibrium is that it is easy to implement and does not require the player to do any calculations.

The case for Bo5 follows by the same argument except that: (1) there are many more cases to analyze in the 3-1 and 3-2 winning situations and (2) the final algebra to show that the expressions coincide are longer and required adding and subtracting multiple terms. But, conceptually, it is exactly the same and you will find that mixing with probability 1/3 makes your opponent indifferent in what deck to queue. You can believe my word, do the algebra yourself, or do it numerically with a computer as some people have already done.

For these manipulations to go through it is important that your opponent uses random.org in all stages. That is, take the case where you win the first game with Deck 1. Your opponent now knows that you will queue Deck 2. If you opponent abandons using random.org and decides to queue his most favorable deck against Deck 1 (because psychologically he/she wants to win now) or, alternatively, he/she decides to queue the unfavorable deck because he/she prefers to get a win with the bad deck first, then the situation and the derivations above change. I did not explore these cases in detail.

My job here is done. Pasca approached two days ago asking me to write something on how queuing decisions on conquest mattered or not. We started with the wrong plan, but thanks to the adamant twitter enthusiasts, we managed to provide an answer that I hope many will find useful and others will surely find obvious. In either case, I hope that at least you value the good intentions that players like Pasca are trying to bring to the Hearthstone community.